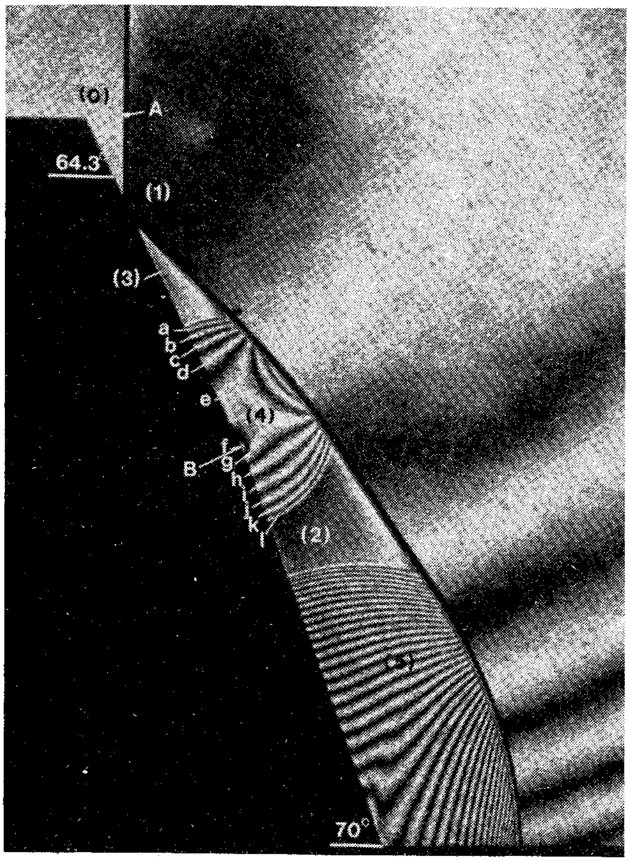

Fig. 2 Infinite fringe interferogram in air. $M_s = 1.39$; $\theta_w = 70$ deg; $p_0 = 750.5$ Torr; $\rho_0 = 0.116 \times 10^{-2}$ g/cm^3; $T_0 = 298.5$ K; $M_2 = 0.26$; $p_2 = 2980$ Torr; $\rho_2 = 0.306 \times 10^{-2}$ g/cm^3; $T_2 = 453$ K; $u_2 = 112.6$ m/s; $M_3 = 0.30$; $p_3 = 2964$ Torr; $\rho_3 = 0.305 \times 10^{-2}$ g/cm^3; $T_3 = 453$ K; $u_3 = 128.7$ m/s; taken at a wavelength of $\lambda = 3471.5$ Å. (u_i stands for the particle velocity in state (i).)

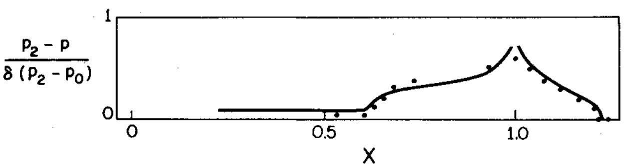

Fig. 3 Comparison of calculated[4] (——) and measured (●) relative pressure deficiency along the surface of the wedge.

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References

[1]Lighthill, M. J., "The Diffraction of Blast. I," *Proceedings of the Royal Society of London*, Ser. A, Vol. 198, 1949, pp. 454-470.

[2]Fletcher, C. H., Weimer, D. K., and Bleakney, W., "Pressure behind a Shock Wave Diffracted through a Small Angle," *Physical Review*, Vol. 78, No. 5, 1950, pp. 634-635.

[3]Srivastava, R. S., "Diffraction of Blast Waves for the Oblique Case," British Aeronautical Research Council Current Papers, No. 1008, 1968.

[4]Srivastava, R. S. and Chopra, M. G., "Diffraction of Blast Waves for the Oblique Case," *Journal of Fluid Mechanics*, Vol. 40, Pt. 4, 1970, pp. 821-831.

[5]Ando, S., "Pseudo-Stationary Oblique Shock-Wave Reflection in Carbon Dioxide-Domains and Boundaries," University of Toronto, UTIAS TN 231, 1981.

[6]Deschambault, R. L. and Glass, I. I., "An Update on Non-stationary Oblique-Shock-Wave Reflections: Actual Isopycnics and Numerical Experiments," *Journal of Fluid Mechanics*, Vol. 131, 1983, pp. 27-57.

Balance of Turbulent Energy in the Linear Wall Region of Channel Flow

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Introduction

A DESCRIPTION of the energy balance in the wall region of turbulent flow is of importance both in understanding the physical processes of turbulent motion and in developing useful closure models.[1] Laufer[2] experimentally determined an energy budget for turbulent pipe flow. Some of the terms in the energy equation were found directly using hot wire probes. Others, in the dissipation term, were obtained using Taylor's hypothesis and the assumption of isotropy; while the term containing pressure was found as the closing entry. Later Townsend[3] corrected Laufer's estimates of the dissipation term and deduced the energy balance given in Fig. 1. The validity of these curves, for small y^+, has not been established conclusively, owing to the breakdown of Taylor's hypothesis and the isotropy assumptions in this region.

Eckelmann[4] and Kreplin and Eckelmann[5] made a detailed study of turbulent oil channel flow with a large viscous sublayer for small y^+ using a hot-film probe and a flush mounted hot-film wall element. In particular, Eckelmann[4] discovered that the rms streamwise and transverse velocities u' and w', respectively, were proportional to y^+ only for a thin region $0 \leq y^+ \leq 0.1$, which he termed the viscous wall layer. In the present Note it is shown that the presence of a very small viscous wall layer is in contradiction with the energy balance in Fig. 1, for at least $0 \leq y^+ \leq 2$, if it is assumed that channel and pipe flows have a similar energy balance in this region.

Turbulent Kinetic Energy Equation

The turbulent kinetic energy equation for fully developed turbulent channel flow is[6]

$$
\begin{aligned}
0 = & \underbrace{-\langle uv \rangle \frac{\partial U}{\partial y}}_{\text{P}} + \underbrace{\frac{\nu}{2}\frac{\partial^2}{\partial y^2}(\langle u^2 \rangle + \langle v^2 \rangle + \langle w^2 \rangle)}_{\text{GD}} \underbrace{- \frac{1}{\rho}\frac{\partial}{\partial y}\langle pv \rangle}_{\text{PD}} \\
& \underbrace{-\nu\Big(\Big\langle\Big(\frac{\partial u}{\partial x}\Big)^2\Big\rangle + \Big\langle\Big(\frac{\partial v}{\partial x}\Big)^2\Big\rangle + \Big\langle\Big(\frac{\partial w}{\partial x}\Big)^2\Big\rangle
+ \Big\langle\Big(\frac{\partial u}{\partial y}\Big)^2\Big\rangle + \Big\langle\Big(\frac{\partial v}{\partial y}\Big)^2\Big\rangle + \Big\langle\Big(\frac{\partial w}{\partial y}\Big)^2\Big\rangle + \Big\langle\Big(\frac{\partial u}{\partial z}\Big)^2\Big\rangle
+ \Big\langle\Big(\frac{\partial v}{\partial z}\Big)^2\Big\rangle + \Big\langle\Big(\frac{\partial w}{\partial z}\Big)^2\Big\rangle\Big)}_{\text{D}} \\
& \underbrace{- \frac{1}{2}\frac{\partial}{\partial y}(\langle vu^2 \rangle + \langle v^3 \rangle + \langle vw^2 \rangle)}_{\text{KD}}
\end{aligned}
\quad (1)
$$

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where $\langle(\ )\rangle$ denotes the time average of ( ); u, v, w are the velocity fluctuations in the streamwise, normal, and spanwise directions, respectively; and U is the local mean streamwise velocity. P represents production, GD gradient diffusion, D "isotropic" dissipation, KD kinetic energy diffusion, and PD pressure energy diffusion. Terms GD+D form $\nu(\langle u\nabla^2 u\rangle + \langle v\nabla^2 v\rangle + \langle w\nabla^2 w\rangle)$, which is the mean work done by the fluctuating part of the shear stress in changing the turbulent kinetic energy.

An analysis of the terms in Eq. (1) for small y^+ may be carried out by expanding them in a Taylor series about $y^+=0$. The following expansions may be derived:

$$\nu\frac{1}{2}\frac{\partial^2}{\partial y^2}\langle u^2\rangle = \frac{u_*^4}{\nu}\left(a^2+3by^+ + \left(\frac{3}{2}c+2d\right)(y^+)^2+\ldots\right) \quad (2)$$

$$\nu\left\langle\left(\frac{\partial u}{\partial y}\right)^2\right\rangle = \frac{u_*^4}{\nu}\,(a^2+2by^+ + (c+d)(y^+)^2+\ldots) \quad (3)$$

$$\nu\frac{1}{2}\frac{\partial^2}{\partial y^2}\langle w^2\rangle = \frac{u_*^4}{\nu}\,(a'^2+3b'y^+ +\ldots) \quad (4)$$

$$\nu\left\langle\left(\frac{\partial w}{\partial y}\right)^2\right\rangle = \frac{u_*^4}{\nu}\,(a'^2+2b'y^+ +\ldots) \quad (5)$$

$$\frac{1}{\rho}\frac{\partial}{\partial y}\langle pv\rangle = \frac{u_*^4}{\nu}\,(ey^+ +\ldots) \quad (6)$$

where velocities have been scaled by the friction velocity u_*, lengths by ν/u_*, and

$$a = (\nu/u_*^2)\langle u_y(0)^2\rangle^{1/2}, \qquad b=(\nu^3/u_*^5)\langle u_y(0)u_{yy}(0)\rangle$$

$$c = (\nu^4/u_*^6)\langle u_{yy}(0)^2\rangle, \qquad d=(\nu^4/u_*^6)\langle u_y(0)u_{yyy}(0)\rangle$$

$$a' = (\nu/u_*^2)\langle w_y(0)^2\rangle^{1/2}, \quad b'=(\nu^3/u_*^5)\langle w_y(0)w_{yy}(0)\rangle$$

$$e = (\nu^2/\rho u_*^5)\langle p(0)v_{yy}(0)\rangle$$

are dimensionless constants where $(\ )_y \equiv \partial(\ )/\partial y$. The remaining terms in Eq. (1) have power series whose leading terms are $\sim(y^+)^2$ or $\sim(y^+)^3$ and, hence, for the present analysis, will be omitted for small values of y^+. An estimate of the range of y^+ for which this assumption is valid will be given subsequently.

Substituting Eqs. (2-6) into Eq. (1) and setting the terms which are constant and those which are linear in y^+ equal to zero gives the identity

$$a^2+a'^2=a^2+a'^2 \quad (7)$$

and the relation

$$e=b+b' \quad (8)$$

A numerical evaluation of Eq. (1) for small y^+ can be carried out using Eqs. (2-6) and Eq. (8) after values for a, b, c, d, a', and b' are determined. These may be obtained by least-square fitting of low degree polynomials to experimental measurements taken by Eckelmann and Kreplin[4,5] of u'/U, w'/U, and $\langle uu_y(0)\rangle/(u'\langle u_y(0)^2\rangle^{1/2})$ in the region $0\le y^+\le 4$ of a channel flow at Reynolds numbers 7700 and 8200. The values of the constants computed in this way are: $a=0.25$, $b=0.027$, $c=0.014$, $d=-0.0089$, $a'=0.065$, and $b'=0.0070$. The details of this procedure may be found in Ref. 7.

Analysis of the Energy Equation

Equation (7) implies that terms GD and −D of Eq. (1) are equal at the wall surface. Derksen and Azad[8] collected experimental measurements of $a^2+a'^2$ for pipe and channel flows (including the data of Eckelmann and Kreplin) and computed an average value of $a^2+a'^2\approx 0.09$. For the present calculations $a^2+a'^2=0.067$.

It may be shown that

$$u'=u_*(ay^+ + (b/2a)(y^+)^2+\ldots) \quad (9)$$

u' varies approximately linearly near the wall as long as the term $(b/2a)(y^+)^2$ in Eq. (9) is significantly smaller than the term ay^+. For the values of a and b computed above, $(b/2a)(y^+)^2$ is equal to, say, 5% of ay^+ when $y^+=0.23$. Thus, for $0\le y^+\lesssim 0.23$, u' varies approximately linearly. For u' to be linear throughout the viscous sublayer $0\le y^+\lesssim 5$, as U approximately is, requires, by a similar argument, a value of $b\approx 0.0012$. This value is a factor of 20 smaller than that computed previously, i.e., 0.027. A similar argument applies to w' and the magnitude of b'. From this it may be concluded

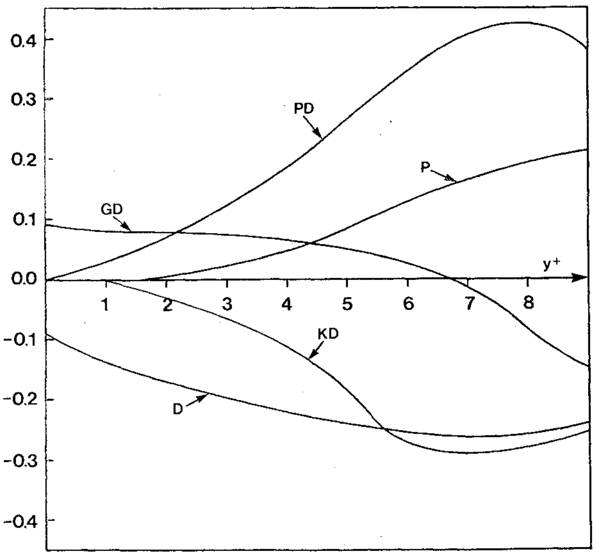

Fig. 1 The energy budget for pipe flow as given by Townsend.[3] Curves P, GD, PD, D, KD correspond to the comparable terms in a modified form of Eq. (1) appropriate to pipe flow. Curve P = production, GD = gradient diffusion, D = isotropic dissipation, KD = kinetic energy diffusion, and PD = pressure energy diffusion.

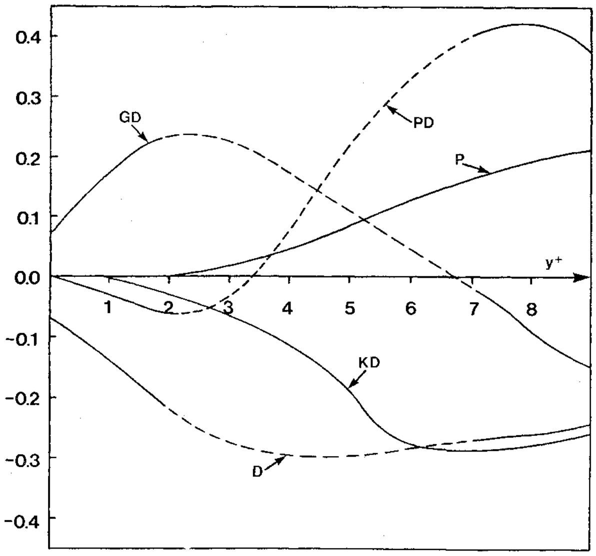

Fig. 2 Modified energy budget for channel (pipe) flow. Curves P, GD, PD, D, KD correspond to the terms in Eq. (1).

that the shortness of the "viscous wall layer" in which $u' \sim y^+$ and $w' \sim y^+$ is responsible for the fact that b and b' are not approximately zero.

Using the values of b and b' computed previously, Eq. (8) gives $e = 0.034$. According to Eq. (6), the magnitude of e is related to the slope of term PD of Eq. (1) at the wall. Since $e > 0$, PD will be negative near the wall. However, it is clear, from the last paragraph that this phenomenon is primarily a consequence of the presence of the viscous wall layer, i.e., if b and b' were much smaller, then e would be smaller and PD would be essentially zero and not negative near the wall.

The nonzero values of b, b', and e imply that next to the wall there exists a region where Eq. (1) reduces to

$$\underbrace{\frac{\nu}{2}\frac{\partial^2}{\partial y^2}(\langle u^2\rangle + \langle w^2\rangle)}_{\text{GD}} \underbrace{- \nu\left(\left\langle\left(\frac{\partial u}{\partial y}\right)^2\right\rangle + \left\langle\left(\frac{\partial w}{\partial y}\right)^2\right\rangle\right)}_{\text{D}} \underbrace{- \frac{1}{\rho}\frac{\partial}{\partial y}\langle pv\rangle}_{\text{PD}} = 0 \tag{10}$$

since the remaining terms are of higher order and therefore negligible. A measure of the extent of this region is the value of y^+ for which the quadratic term in Eq. (2) becomes equal to, say, 5% of the magnitude of the first two terms. A computation gives $y^+ \approx 2.1$. It thus may be concluded that Eq. (10) holds approximately for $0 \leq y^+ \lesssim 2.1$.

Equation (1) applies specifically to channel flow, although with slight modification it also holds for a pipe flow (see Ref. 6, p. 735). In the region considered in Fig. 1 ($0 \leq y^+ \leq 9$) it may be assumed that the terms in Eq. (1) and their equivalents for pipe flow have similar numerical values if scaled by velocity u_* and length ν/u_*. Thus the values of the terms in Eq. (1) for $0 \leq y^+ \lesssim 2$ which have been determined by the present study may be plotted so as to merge smoothly into the values of their equivalents for pipe flow as given in Fig. 1.

A plot of the revised energy budget is given in Fig. 2. The solid part of curves GD, D, and PD for $0 \leq y^+ \lesssim 2$ represent the values computed from Eqs. (2-6) using the constants given previously. The solid curves for $y^+ \gtrsim 7$ are the values given by Townsend[3] as shown in Fig. 1. The dashed lines represent possible forms of these terms for $2 \lesssim y^+ \lesssim 7$ which provide for a smooth transition between the two parts. For the purposes of comparison, curves P and KD from Fig. 1 have been included in Fig. 2. The curves in Fig. 2 are in essentially exact energy balance for $0 \leq y^+ \lesssim 2$ and $y^+ \gtrsim 7$. Since the positions of curves GD, D, and PD are not known precisely in the region $2 \lesssim y^+ \lesssim 7$, no attempt has been made to force exact energy conservation here.

The energy budget in Fig. 2 has several significant differences from that in Fig. 1. In particular, the pressure energy diffusion term, curve PD, is negative near the boundary, in contrast to the strictly positive values given by Townsend.[3] Hinze (Ref. 6, p. 325) gives an argument implying that $\langle p(\partial v/\partial y)\rangle > 0$ if $2\langle u^2\rangle > \langle v^2\rangle + \langle w^2\rangle$. This condition is satisfied at every point in a channel flow. Expanding $\langle p(\partial v/\partial y)\rangle$ in a Taylor series about $y = 0$ gives

$$\frac{1}{\rho}\left\langle p\frac{\partial v}{\partial y}\right\rangle = \frac{u_*^4}{\nu}(ey^+ + \ldots) \tag{11}$$

Since $e > 0$ as determined by the present study, it is evident that for small y^+, $\langle p(\partial v/\partial y)\rangle > 0$, in agreement with Hinze. Thus a negative value for term PD at small y^+ is consistent with the expected behavior of $\langle p(\partial v/\partial y)\rangle$. A negative value of PD near the wall is also consistent with the original energy budget given by Laufer[2] and, in addition, may be inferred from the analysis of Rotta.[9]

Two other significant differences between Figs. 1 and 2 are the greater level of dissipation indicated by curve D and the fact that the energy diffusion term, GD, achieves a local maximum in the interior of the fluid at $y^+ \approx 2.3$, and not at the wall surface.

According to the revised energy budget, the region $0 \leq y^+ \lesssim 2$ is devoid of significant turbulence production. It acts as an energy sink in which all of the turbulent kinetic energy that diffuses into it, indicated by curve GD, is lost. Two mechanisms of energy loss are implied in Fig. 2. Curve D represents the loss of turbulence energy by its direct conversion to heat. Curve PD accounts for the work done by the fluctuating pressure field in retarding the motion of fluid particles. This interpretation of curve PD follows from the identity

$$-\frac{1}{\rho}\frac{\partial}{\partial y}\langle pv\rangle = -\frac{1}{\rho}\left(\left\langle u\frac{\partial p}{\partial x}\right\rangle + \left\langle v\frac{\partial p}{\partial y}\right\rangle + \left\langle w\frac{\partial p}{\partial z}\right\rangle\right) \tag{12}$$

in which the right-hand side is the primitive form of the pressure term in the energy equation (Ref. 6, p. 373). The deceleration of fluid particles near the boundary implied by term PD is consistent with some of the widely held views, (e.g., Ref. 10, p. 242 et seq.) concerning the dynamics of coherent structures in turbulent boundary layers. In particular, it is believed that a deceleration of fluid particles by the pressure field initiates the occurrence of the burst sequence.

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References

[1]Chien, K.-Y., "Predictions of Channel and Boundary Layer Flows with a Low-Reynolds-Number Turbulence Model," *AIAA Journal*, Vol. 20, Jan. 1982, pp. 33-38.

[2]Laufer, J., "The Structure of Turbulence in Fully Developed Pipe Flow," NACA TR 1174, 1954.

[3]Townsend, A. A., *The Structure of Turbulent Shear Flow*, 1st Ed., Cambridge University Press, London, 1956.

[4]Eckelmann, H., "The Structure of the Viscous Sublayer and the Adjacent Wall Region in a Turbulent Channel Flow," *Journal of Fluid Mechanics*, Vol. 65, Sept. 1974, pp. 439-459.

[5]Kreplin, H.-P. and Eckelmann, H., "Behavior of the Three Fluctuating Velocity Components in the Wall Region of a Turbulent Channel Flow," *Physics of Fluids*, Vol. 22, July 1979, pp. 1233-1239.

[6]Hinze, J. O., *Turbulence*, 2nd Ed., McGraw-Hill Book Co., New York, 1975.

[7]Bernard, P. S. and Berger, B. S., "The Balance of Turbulent Kinetic Energy in the Wall Region of Channel Flow," Dept. of Mechanical Engineering, Univ. of Maryland, Technical Rept. 82-1, 1982.

[8]Derksen, R. W. and Azad, R. S., "Behavior of the Turbulent Energy Equation at a Fixed Boundary," *AIAA Journal*, Vol. 19, Feb. 1981, pp. 238-239.

[9]Rotta, J. C., "Turbulent Boundary Layers in Incompressible Flows," *Progress in Aeronautical Science*, Vol. 2, Pergamon Press, New York, 1962, pp. 1-220.

[10]Willmarth, W. W., "Structure of Turbulence in Boundary Layers," *Advances in Applied Mechanics*, Vol. 15, 1975, pp. 159-254.